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Mathematical modeling to optimize production planning and scheduling in a small foundry with multiple alternating furnaces

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RESUMO

Propósito – Este trabalho apresenta uma extensão a um modelo da literatura para o dimensionamento e sequenciamento de lotes em uma fundição de pequeno porte com múltiplos fornos alternados. O objetivo do modelo é minimizar custos de atraso e estoque. Além disso, busca-se o melhor aproveitamento da capacidade de carga dos fornos.
Design/método/abordagem – Modelagem matemática é apresentada para o problema de dimensionamento e sequenciamento de lotes em uma fundição de pequeno porte. Dados oriundos das carteiras de pedidos da empresa foram coletados e questionários de validação do modelo foram aplicados.
Resultados – O modelo estendido foi capaz de gerar bons planos de produção em diferentes horizontes de planejamento, com desempenho melhor que os atuais métodos obtidos pela empresa.
Originalidade/valor – a extensão do modelo contribui com a literatura por abordar a existência de múltiplos fornos não simultâneos, característica pouco explorada até então. Uma comparação com outros modelos é realizada para indicar um modelo mais adequado para uma aplicação real.
Palavras-chave: Dimensionamento de lotes. Fundição. Programação inteira mista. Sequenciamento de ligas.

ABSTRACT

Purpose *- This study presents an extension to a model in the literature for lot-sizing and scheduling in a small foundry with multiple alternate furnaces. The purpose of the model is to minimize delays and inventory costs. In addition, it determines the best use of the load capacity in the furnaces.*
Theoretical framework *– Lot-sizing in foundries in the marketplace is a subject of academic interest due to its applicability and mathematical and computational complexity. Many papers address the production problem in foundries with a single furnace, however, few papers address the possibility of multiple furnaces.*
Design/methodology/approach *- Mathematical modeling was used to represent the lot-sizing and scheduling problem in a small foundry. Data from the company's order books were collected and model validation questionnaires were applied.*
Findings *- The extended model was able to generate good production plans at different planning horizons, with better performance than the current methods obtained by the company.*
Originality/value *- the extension of the model contributes to the literature by addressing the existence of multiple non-simultaneous furnaces, a feature that has not been greatly explored. A comparison with other models is performed to indicate the most suitable model for actual application.*
Keywords*: Alloys scheduling. Foundry. Lot size. Mixed integer programming.*

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1. INTRODUCTION

The casting process is based on the production of items made of metal alloys. The foundry industry is a primary company that supplies intermediate goods to other institutions, such as automotive, steel, construction, and others.

For companies meeting market demand within an appropriate time window, it is essential to have good Production Planning and Control (PPC). The function of PPC is to schedule and control production so that a company meets production requirements as efficiently as possible. Decisions made at the PCP are hierarchical: i) strategic planning: long-term planning, related to the highest level of the company, determining the company's global goals; ii) tactical planning: medium-term planning, responsible for the correct and timely use of the company's available resources; and iii) operational planning: short-term planning, that determines the decisions made in the company's daily routine (BONNEY, 2000).

For Drexl and Kimms (1997), planning and scheduling production efficiently is one of the biggest challenges for company managers. In this context, the lot-sizing problem consists of determining, within a finite time horizon, in which periods there should be production and the number of items to be produced to meet their demands, subject to capacity constraints and minimizing total costs (BRAHIMI *et al.*, 2006).

The lot sizing and scheduling problem in a foundry can be seen as the search for a production plan that determines which alloys should be cast for manufacturing many items respecting the company's production constraints. This plan must minimize costs, such as furnace preparation, inventory maintenance, and order delays (CAMARGO; MATTIOLI; TOLEDO, 2012).

Due to their importance in supplying parts to various industry sectors and market pressures (including competition and economic risks), foundries need to improve themselves to meet demands and profit from their operations. This improvement may arise from the technological modernization and/or technique refinement for process optimization to extract the maximum from the capacity installed. Due to their nature (usually small and medium-sized companies with a large variety of products), market foundries face several difficulties in improving their performance because of the number of variables involved in the planning process and structural limitations.

Most of the literature deals with single furnace foundries. Despite the importance of using multiple furnaces common in foundries, few studies have addressed this issue. Silva and Morabito (2004) address the issue of multiple parallel furnaces. Toledo *et al. (*2014) proposed a mixed integer programming model for multiple alternating furnaces. Thus, it is important to treat multiple furnaces as this is a relevant point for foundries. Furthermore, Clark, Almada-Lobo, and Almeder (2011) point out the intrinsic relationship between lot-sizing and scheduling in process industries. The authors emphasize the need for production planning that considers both decisions simultaneously for efficient use of resource capacity to achieve planning with the appropriate use of mathematical modeling.

In this paper, we propose an extension to a lot-sizing and scheduling model found in the literature. The model considers the use of multiple alternating furnaces, filling a gap in the literature. A comparison of results (objective function value, optimality gap, and sub utilization of the furnaces) of the extended model with other literature models contribute to indicating the one that may present the best performance for the production environment studied. In addition, the model will be applied in a real foundry. The validation technique proposed by Oral and Kettani (1993) is used to verify if the proposed model is adequate for the production planning of a market foundry. As a result, a production plan of items and alloy sequencing in the furnaces is obtained to support decision-making at the operational and tactical levels.

This paper is organized as follows: Section 2 presents the main works found in the literature on lot-sizing and scheduling in foundries. Section 3 introduces the methodological procedures used in this work. Computational tests performed for validation and application of the model in foundries are described in Section 4. A discussion of the results is presented in Section 5. Finally, conclusions and indications for future work are provided in Section 6.

2. THEORETICAL FOUNDATION

In Brazil, most cast products are sold to automotive and steel industries, produced in captive foundries, which are departments of large companies. In these foundries, production is serial, automated, and oriented to supply internal needs. A small portion of the cast products is left for other sectors such as the mechanical and infrastructure industries, and this small demand is met by small and medium-sized foundries, known as market foundries. The market foundries are usually characterized by relatively low demand and little organized management structure, due to the absence of an adequate market policy (CAMARGO; MATTIOLLI; TOLEDO, 2012), (ARAUJO; ARENALES, 2003).

The manufacturing process of a foundry can be summarized as follows: shape, internal cavities, and external fittings are made in the cores. Molding is the making of the molds that shape the outer parts of the piece. The molds and cores are combined into a single set of parts to receive the molten metal in the pouring stage. Melting transforms raw materials such as aluminum, pig iron, and other metal alloys into liquid metal. Once the molten metal and the cores and molds are ready, the casting process begins, in which the molten metal alloy is poured into the core mold assembly, filling all the cavities of the assembly. After a cooling time, the alloy solidifies, and the part is removed from the mold. The next step is deburring, where the part goes through the finishing processes.

The lot-sizing problem in foundries has been widely studied due to its economic and academic importance. Mathematical models and solution methods for lot-sizing in foundries with a single furnace can be found in Santos-Meza and Oliveira (2002), Araujo and Arenales (2003), Araujo, Arenales and Clark (2004), Teixeira-Jr, Fernandes and Pereira (2006), Araujo *et al.* (2008), Tonaki and Toledo (2010), Camargo *et al.* (2012), Basiura *et al.* (2015), Stawowy and Duda (2017), Duda and Stawowy (2018). In these studies, mixed-integer linear programming models seek to minimize furnace setup costs (configuration changes), inventory costs, and item backlogs to determine a production plan for the alloys used in the furnace and the items to be produced are proposed. As solution methods, the authors use commercial solvers such as CPLEX, and different heuristics and meta-heuristics are proposed.

Furtado et al. (2019) also address the lot-sizing and scheduling problem in market foundries with a single furnace, considering the orders to which the items belong. In addition to the traditional aspects addressed in other studies, the models proposed by the authors aim to minimize the cost delay in the delivery of orders with the possibility of partial deliveries.

Li *et al.* (2017) present a mathematical profit maximization model for production planning in a market foundry with a flow shop system and limited capacity. Four different types of costs are considered in the model: material costs, process costs, machine utilization costs, and delay costs. A solution method based on a genetic algorithm is presented to solve the real problem.

Models with multiple furnaces, aiming to minimize setup costs, inventory, and item backlog are found in Silva and Morabito (2004) and Stawowy and Duda (2020), which consider production lines in which the furnaces can be used in parallel, i.e., simultaneously. An approach using multiple non-simultaneous furnaces is proposed by Toledo *et al.* (2014), who present a mixed integer programming model for multiple furnaces in a foundry located in the interior of São Paulo. A linear programming model with multiple kilns and an objective function to maximize the average efficiency of the kilns is presented and solved by Park (2013).

It can be observed that the lot-sizing in market foundries is a subject of academic interest due to its applicability and mathematical and computational complexity. Many studies address the production problem in foundries with a single furnace; however, few works address the possibility of multiple furnaces. Thus, aiming to contribute with studies in this area and being a real problem of a foundry, this paper proposes a mathematical model that considers the possibility of the non-simultaneous use of multiple furnaces. In addition, a comparison of this model with models found in the literature will be performed in relation to the result of the objective function and under-utilization of the furnaces to determine the best model to be applied in a real case.

3. METHODOLOGICAL PROCEDURES

Modeling is a quantitative method that represents a production system in mathematical and computational language, using analytical techniques to determine values in a production system. This work is based on the Quantitative Normative Axiomatic Research methodology, common in Operations Research investigations. The research is called quantitative axiomatic because it is primarily oriented to idealized problem models and is normative because it is based on models that prescribe a decision for the problem (MORABITO; PUREZA, 2010; BERTRAND; FRANSOO, 2002). There is also an empirical part of the research as this work was done by analyzing a real problem.

This section details the production environment studied and the methodological procedures used to address the problem of lot-sizing with multiple alternating furnaces in a small foundry.

3.1 Study object

This study is based on a Brazilian market foundry located in the Triângulo Mineiro. The active structure of this foundry has a three-cavity furnace with nominal capacities of 800, 800, and 400 kilograms (kg). Due to restrictions in the contracted energy demand, the cavity furnaces do not operate simultaneously, restricting the melting capacity to 800 kg per melt. This structure ensures that using a multiple crucible furnace has the same behavior and restrictions for lot-sizing and alloy sequencing as a multi-furnace foundry.

The foundry works with a make-to-order system, producing only items that make up the order portfolio. Due to the great variety of products, all production is programmed based on orders released by the commercial sector. Each order may contain several items and different delivery dates, according to the customer's request. These factors may imply the fractioning of the order according to each item's available material and delivery date. Respecting the material constraint and delivery date, the PPC sector groups items from several orders to try to use the maximum capacity of each furnace.

For better use of resources and cost reduction, the company forces the use of the maximum capacity of the 800 kg furnace. Using the 400 kg furnace takes place in two situations: when parts weighing more than 800 kg and less than 1200 kg are produced, or if the demand for items using the same alloy does not exceed 400 kg. In the first case, the 800 kg crucible is used and then the 400 kg cavity until the complete melting of the material to be filled by the two crucibles sequentially. This process requires twice the casting time due to the need to alternate the furnaces. The foundry works with nodular and gray metal alloys. The composition of each alloy is given in Table 1.

Table 1 - Cast Materials.

Material	Description	Raw material	Approximate composition
60-45-12	Nodular	Pig Iron	48.5%
		Nodular return 60-45-12	24.5%
		Steel Scrap	24.5%
		Special alloys and fuels	2.5%
70-50-05	Nodular	Pig Iron	48.5%
		Nodular return 70-50-05	24.5%
		Steel scrap	24.5%

		Special alloys and fuels	2.5%
80-60-03	Nodular	Pig Iron	48.5%
		Nodular return 80-60-03	24.5%
		Steel Scrap	24.5%
		Special alloys and fuels	2.5%
A48CL30	Gray	Scrap iron	67.0%
		Steel scrap	15.0%
		Gray return	15.0%
		Special alloys and fuels	3.0%
A48CL30S	Special Gray	Scrap iron	67.0%
		Steel scrap	15.0%
		Special gray return	15.0%
		Special alloys and fuels	3.0%

Source: Authors (2021).

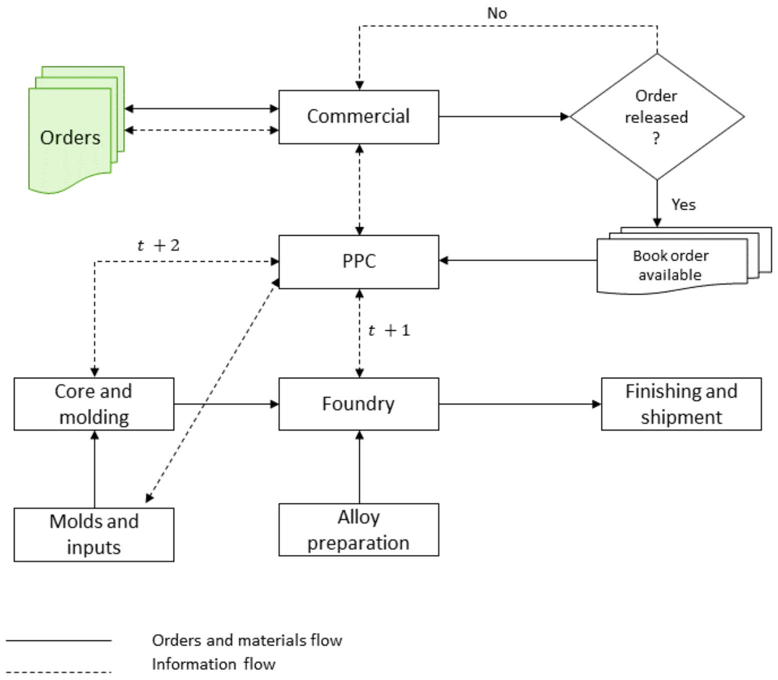
As illustrated in Table 1, each alloy has at least one component (raw material) different from other alloys. For this reason, each furnace should contain only one type of alloy to be melted.

Another important characteristic for the company is alloy melting temperature, according to the project to be cast. Parts with narrow ducts need a less viscous alloy to fill them, requiring a higher melting temperature. On the other hand, large parts can be melted at lower temperatures, reducing energy costs. Even though this work does not consider the exact melting temperature cost, the company adopts this premise and alters the entire production plan. To ensure the integrity in the comparison of the models, the strategy to avoid the technical issue of melting temperatures for the same alloy was to consider such different alloys. The 60-45-12 alloy, for example, may contain parts that require three different temperatures. Thus, we can assume up to three different alloys for the same material.

Currently, the production planning is done two days in advance for the mold and core sectors (responsible for manufacturing molds and complements) and one day in advance for the foundry sector. This implies that the expected production for the two-day horizon is known except for some special cases. The exceptions are limited to parts with some constraint, such as a lack of mold, raw material, project, etc., or when the part urgently enters the portfolio, ensuring priority in its production.

The planning process begins after the orders are received and are validated by the commercial sector. The available order book is organized according to the due date for delivery of each order or item (when there are different due dates in the same order). The PPC is responsible for organizing the orders and creating the production orders. The mold and core sectors prepare the models to be casted. The foundry sector is responsible for melting the alloys and filling the molds. Finally, the parts are finished and available for invoicing and shipping to customers. Figure 1 illustrates the process described above until the casting and shipping of the part, highlighting the flow of information exchanged between the PPC and other areas involved in the production process. This factor is fundamental to synchronize the planning according to the available resources.

Figure 1 - Flowchart of the casting process.



Source: Authors (2021).

Briefly, the casting process in the analyzed company follows the sequence below:

1. The PPC receives the orders validated by the commercial area (book order available).

2. Planning begins for what will be cast two days ahead. The planning process consists of grouping the items by the delivery date and alloy type, prioritizing the most overdue items, and considering the melting capacity of the furnace.

3. After grouping the items, production orders are created until the casting capacity is completed in one day.

4. Once the production orders are completed, the orders are forwarded (at the end of the day or the beginning of the next day) to the core shop and mold-making department to start making the molds and cores.

5. In parallel, one lane of the production orders is used for the alloy preparation. The alloy preparation consists of weighing and grouping all the metallic material that will compose each batch the next day in boxes prepared for the furnace loads.

6. Two days after planning, the casting of the generated orders begins. The process consists of heating the furnace and melting each case of material prepared the day before. After the required time, the liquid formed is poured into the previously prepared molds.

7. Next, the parts are demolded and given the necessary finishing touches to make them available for the billing and shipping sectors.

3.2 Mathematical model

Araujo, Arenales and Clark (2004) present a mathematical formulation to represent the production planning problem in foundries with a single furnace. This section presents the proposed model for the production planning of alloys with multiple alternate furnaces in a small foundry. This model is an extension of the work proposed by these authors and will hereafter be called Multiple Alternating Furnace (MAF1) model. The indexes, input parameters and decision variables of the (MAF1) model are presented in Table 2:

Table 2 - Input parameters and decision variables of the Multiple Alternating Furnace model.

Indices	
$k = 1, \cdots, K$:	Alloys
$i = 1, \cdots, N$:	Items
$t = 1, \cdots, T$:	Periods
$\eta = 1, \cdots, NS$:	Micro-periods
$m = 1, \cdots, M$:	Furnaces (machines)
Parameters	
d_{it}:	Number of items i ordered per period t
Cap_m:	Furnace capacity m per micro-period (kg).
p_i:	Gross weight (kg) of item i.
b_{it}:	Penalty for delaying a unit of item i in period t
h_{it}:	Penalty for holding a unit of item i in period t
$S(k)$:	Set of items i that use alloy k
s_{km}:	Penalty for preparation for alloy k, in furnace m.
Decision Variables	
$X_{im\eta}$:	Number of items i produced in furnace m, in micro-period η .
I_{it}:	Number of items i held at the end of period t
B_{it}:	Number of items i delayed at the end of period t
$Y^k_{m\eta}$:	1, if there was preparation for alloy k, in furnace m, in micro-period η; 0, otherwise.
$Q^k_{m\eta}$:	1, if furnace m is prepared for alloy k, in micro-period η; 0, otherwise.

Source: Authors (2021).

Multiple Furnace Alternating Model:

$$min \sum_{i=1}^{N}\sum_{t=1}^{T}(b_{it}B_{it} + h_{it}I_{it}) + \sum_{\eta=1}^{L}\sum_{k=1}^{K}\sum_{m=1}^{M} s_{km}Y_{m\eta}^{k} \qquad (1)$$

S.t:

$$I_{i,t-1} - B_{i,t-1} + \sum_{\eta=1}^{L}\sum_{m=1}^{M} X_{im\eta} + B_{it} = d_{it} + I_{it} \qquad i = 1,\cdots,N, \quad t = 1,\cdots,T \qquad (2)$$

$$\sum_{i\in S(k)} p_i X_{im\eta} \leq Cap_m\, Q_{m\eta}^{k} \qquad k = 1,\cdots,K \;\; m = 1,\cdots,M \;\; \eta = 1,\cdots,L \qquad (3)$$

$$Y_{m\eta}^{k} \geq Q_{m\eta}^{k} - Q_{m(\eta-1)}^{k} \qquad k = 1,\cdots,K \;\; m = 1,\cdots,M \;\; \eta = 1,\cdots,L \qquad (4)$$

$$\sum_{k=1}^{K}\sum_{m=1}^{M} Q_{m\eta}^{k} \leq 1 \qquad \eta = 1,\cdots,L \qquad (5)$$

$$Q_{m\eta}^{k} \in \{0,1\} \qquad k = 1,\cdots,K \;\; m = 1,\cdots,M \;\; \eta = 1,\cdots,L \qquad (6)$$

$$Y_{m\eta}^{k} \geq 0 \qquad k = 1,\cdots,K \;\; m = 1,\cdots,M \;\; \eta = 1,\cdots,L \qquad (7)$$

$$X_{im\eta}, B_{it}, I_{it} \geq 0 \text{ and integers} \qquad i = 1,\cdots,N, t = 1,\cdots,T, m = 1,\cdots,M \qquad (8)$$

The objective function (1) minimizes inventory, item delay, and furnace setup costs. Constraints (2) represent the balance of production and inventory of the items according to the demand. Inequalities (3) represent the capacity constraints (kg) of the furnace. The total amount of alloy to be melted must respect the total capacity of the furnace to which it has been allocated. Constraints (3) also ensure that the furnace is prepared to produce items of the same alloy. According to the alloy to be melted, the existence of furnace configuration changes is presented in constraints (4). Inequalities (5) ensure that at most a single furnace is prepared for a single alloy in each micro-period. The binary variable definition indicating furnace preparation and its initial period is given by constraints (6). Finally, in (7) and (8), we have the non-negativity and completeness constraints of the variables.

3.3 Model evaluation

The model evaluation compares the main aspects and differences between the proposed model and the models proposed by Araujo, Arenales and Clark (2004) and Toledo *et al.* (2014). Moreover, they verify which one best fits the data of the foundry studied. The models were solved by CPLEX 12.6.1 optimization software and the values obtained for objective function, optimality gap and underutilization of the furnace were compared.

We used 19 instances found in the literature. The instances named from 1 to 11 were proposed by Tonaki and Toledo (2010) and the instances from 12 to 19 appear in the work of Camargo and Navarenho (2016). Table 3 shows the main characteristics of the instances adopted in the evaluation tests.

Table 3 - Characteristics of the instances.

Instances	**Alloys**	**Items**	**Periods (days)**	**Capacity (Kg)**	**Demand (Kg)**	**Description**
1	5	165	3	11400	13863.9	Delayed items from the five most frequent alloys.
2	5	165	5	19000	13863.9	Delayed items from the most frequent five alloys.
3	5	228	3	11400	20139.45	All items from the five most frequent alloys.
4	5	293	5	19000	24040.45	All items from the five most frequent alloys.
5	16	225	3	11400	17211.3	All delayed items.
6	16	225	5	19000	17211.3	All delayed items.
7	16	224	3	11400	15441.5	Approximately 90% of the items are delayed.
8	16	224	5	19000	15441.5	Approximately 90% of the items are delayed.
9	15	224	3	11400	13731.3	Approximately 80% of the items are delayed.
10	15	224	5	19000	13731.3	Approximately 80% of the items are delayed.

11	19	383	5	19000	29311.95	Complete book order.
12	19	150	5	19000	11990.05	Items without delays and up to four days delivery time.
13	8	199	5	19000	18129.05	Items without delays and up to four days delivery time.
14	8	283	5	19000	23442.95	Items without delays and up to four days delivery time.
15	8	383	5	19000	23442.95	Items without delays and up to four days delivery time.
16	19	199	5	19000	18129.05	Items without delays and up to four days delivery time.
17	19	283	5	19000	23442.95	Items without delays and up to four days delivery time.
18	19	383	5	19000	29311.95	Items without delays and up to four days delivery time.
19	8	150	5	19000	11990.05	Items without delays and up to four days delivery time.

Source: Authors (2021).

It can be seen in Table 2 that instances 1 to 11 present many delayed items, representing common scenarios in foundry industries. Instances 12 to 19, on the other hand, do not present delayed items. A particularity in the scenarios that start with many delayed items is camouflaging the furnace preparation costs. These costs become insignificant when compared to scenarios without delayed items. However, when considering a longer planning horizon and constantly updating the backlog items, there is a tendency to reduce the delay and increase the setup and furnace changeover costs, making this decision to be better analyzed. In other words, as the delay is reduced, the furnace setup becomes more significant in the decision-making process. Since the MFA1 model is a deterministic model, all input data are known a priori. Thus, in each instance represented in Table 2 all demanded items are known, as well as the specific alloy to be used and the weight (kg) of each item. The MFA1 model aims to meet the demand of the items, minimizing costs related to delays and inventory of the items and setup costs of the furnaces. The cost parameters for the tested instances were calculated as proposed by Araujo and Arenales (2003), as explained below:

Let them be:

β_i: Number of delayed periods for item i

γ_i: Number of anticipation periods of item i

The backlog of an item suffers a penalty $b_{it} = p_i \cdot \beta_i$. The inventory cost of an item i is calculated by $h_{it} = (p_i \cdot \gamma_i)/10$. The cost definition allows the model to prioritize the production of backlog items, without disregarding costs inherent in the inventories of items.

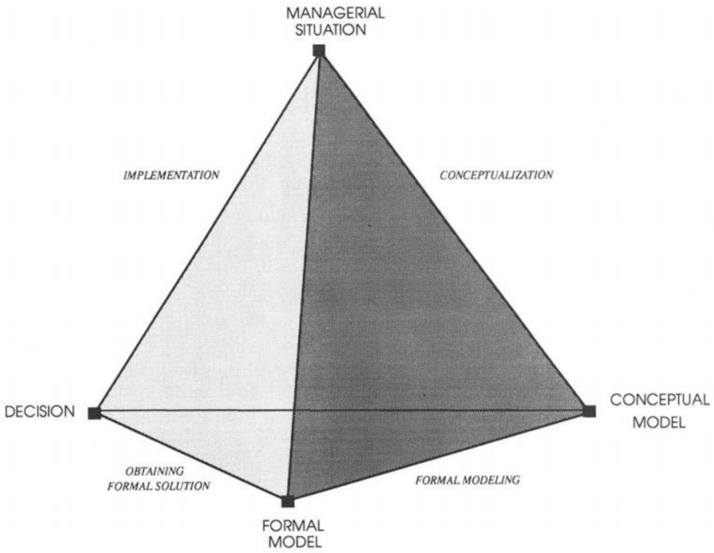
3.4 Model validation

After carrying out computational tests with the instances shown in Table 2, which model could be applied to plan alloy production in the foundry studied was defined. Fachini et al. (2017) define validation of a model as the confirmation of the representativeness of the real scenario. The authors cite the comparison between the model result and that obtained by the company, collected in historical data, as an effective form of validation. However, this method is highly influenced by the quality of the defined parameters and model input data. Using consolidated methods to validate Operations Research (OR) problems is necessary to use the model on its merits and unbiased. Oral and Kettani (1993) present a tetrahedron-shaped model validation framework with its facets and vertices: managerial, conceptual model, formal model, and decision. The authors state that most OR problems can be characterized with only three of the four vertices mentioned, forming one of the facets below:

- Descriptive facet: formed by the vertices Management situation, Conceptual model, and the Formal model. The focus is to understand the system or the management situation in which it is inserted.
- Theoretical facet: formed by the stages Conceptual Model, Formal Model, and Decision. This facet seeks the construction of a formal model and solution methods that accurately represent the theoretical model.
- Prototype facet: in this facet, the "Conceptual model" element is in the background. This facet deals with models that are already known and consolidated. The focus of this facet is to apply a formal model to assist in decision-making to solve a managerial situation.

Figure 2 demonstrates the tetrahedron formed by the vertices and facets of this validation process.

Figure 1 – The quartet of the modeling validation process.



Source: Oral and Kettani (1993).

According to Oral and Kettani (1993), the validation of an OR model follows some practices according to the facet where the problem fits. For each facet, there are validations necessary for the acceptability of the problem. Table 4 presents the types of validations by facets proposed by the authors.

Table 4 - Facets and validations

Facet / Validation	Prototype	Pragmatic	Descriptive	Theoretical
Formulation	x			
Legitimation		x		
Fitness			x	
Verifying				x
Experimental	x			x
Operational	x	x		
Conceptual		x	x	
Logic			x	x
Data	x	x	x	x

Source: adapted from Oral and Kettani (1993).

This work deals with the proposition of a model to help decision-making that will directly influence an organization's bottom line. According to the previous descriptions, the proposed model is a problem that fits the Prototype Facet. Based on Table 4, the validation steps were defined and applied to the foundry analyzed.

3.4.1. Formulation validation

The MFA1 model extends the general lot-sizing problem by Araujo, Arenales and Clark (2004). This model and extensions have been published and disseminated in works with consolidated results, including the planning of small foundries and safely allowing the validation of the formulation. The adaptations of the MFA1 model do not alter the model's structure, corroborating the guarantee of the model's validity.

3.4.2. Data validation

Crucial to the success of this work was the generation of planning solutions that can be applied in practice even if they do not guarantee the best possible result. Real data was collected, and the company's PPC consolidated the input data.

Besides the input data, the process parameters that interfere in the proposed formal model were validated to ensure the minimum interference by parameter inconsistency. The parameters adopted in the literature were accepted by those involved in the planning stage and used due to the lack of actual knowledge of inventory, delay, and setup costs. Thus, the data and parameters used are valid.

3.4.3. Experimental validation

For this validation step, the plans generated from the MFA1 model with real data, the current production plan practiced by the company, and a verification questionnaire were considered. The comparison of the two plans was the basis for this validation, as well as a questionnaire designed for those responsible for the PCP and for the company's production.

The quality of the solution obtained by the model and the perception of the users validate the experiment.

Thus, the questionnaire for experimental validation focuses on the quality criteria exposed by Oral and Kettani (1993). The criteria are the level of perception obtained after knowing the model, sensitivity to changes in model parameters, acceptability level, applicability level, and usefulness level of the result generated.

To assess the model's quality, Questionnaire 1 (Appendix 1) was distributed to the person responsible for production planning (technical level), to the person responsible for the technical department (Mechanical Engineer), and to the planning trainee (Mechanical Engineer). The interviewee answered the questions according to the scale: 1 - I don't agree; 2 - I agree with restrictions and 3 - I agree completely. Complementary information given by the respondents are allowed.

3.4.4. Operational validation

Unlike experimental validation, operational validation is based on five main attributes related mainly to the executability of the formal model and its impact on the operation (ORAL AND KETTANI, 1993). These are: usability of the formal model, the real usefulness of the formal model, time to obtain the solution, the synergy of the result with the previous decisions, and the cost of implementing the model in practice.

The validation of this step is the biggest barrier encountered mainly, due to the cost involved in implementing a PCP with an optimization tool. For this step, a questionnaire, Questionnaire 2 (Appendix 2), was developed for validation (distributed to the people responsible for planning) and comparisons between the real production plans and the one generated by the model were made. The same validation scale used for Experimental Validation was used for this Questionnaire.

4. RESULTS

4.1. Model evaluation

Computational tests were performed to find the best performance of the models proposed by Araujo, Arenales and Clark (2004), Toledo *et al*. (2014) and MFA1 using commercial optimization packages (CPLEX version 12.6.1). All tests were performed by running the instances presented in Table 2 for 3600s on a computer with an Intel Core I3-5005U CPU 2GHz processor with 4 GB of RAM.

Table 5 presents the results obtained with CPLEX for the objective function (solution cost) and optimality gap. The smallest values for the solution cost and gap are highlighted in bold.

Table 5 - Objective function and optimality gap for instances 1 to 19**.**

Instance	Araújo, Arenales and Clark		Toledo et al. (2014)		MFA1	
	Solution cost	Gap	Solution cost	Gap	Solution cost	Gap
1	157232.75	0.58%	157226.25	0.72%	**157126.45**	**0.53%**
2	157665.05	0.79%	157630.85	1.04%	**157423.75**	**0.74%**
3	184048.91	**0.55%**	**183823.48**	0.64%	184195.73	0.74%
4	217345.70	2.30%	**215936.65**	**1.66%**	216087.85	1.71%
5	323053.10	1.20%	324733.60	2.04%	**322834.70**	**1.07%**
6	**359734.40**	**2.43%**	360682.40	3.34%	359862.80	2.62%
7	285478.80	1.16%	286306.00	1.91%	**285289.65**	**1.14%**
8	**303727.10**	**2.30%**	305724.25	3.65%	306131.10	3.08%
9	**249764.20**	**1.36%**	250603.25	2.13%	250399.60	1.70%
10	256588.40	3.69%	256301.55	4.07%	**253281.00**	**2.35%**
11	474126.54	2.05%	474534.26	2.99%	**473743.84**	**2.01%**
12	**14153.66**	**17.53%**	14420.44	33.57%	14311.06	26.71%
13	4072.48	**50.34%**	**3985.69**	68.50%	4279.21	69.63%
14	17571.65	**40.21%**	17349.11	41.09%	**17079.04**	40.68%
15	49001.19	10.84%	49837.66	12.44%	**47187.44**	**7.50%**
16	7153.50	**18.90%**	8484.10	48.78%	**7102.19**	25.38%
17	**32148.50**	**4.74%**	33626.69	14.11%	32440.18	6.23%
18	79796.05	**6.44%**	**79128.32**	10.80%	79299.69	6.70%
19	3666.13	**17.81%**	3309.70	31.60%	**3241.13**	27.07%

Source: Authors (2021).

The results of waste in the furnace are presented in Table 6, in kilograms and as a percentage of waste. Waste indicates the volumes and the percentages of unfilled furnaces. In practice, waste represents extra rework, maintenance, and energy resources. Even though it is not directly in the objective function, the underutilization was analyzed because it represents an important factor for the foundry. This result refers to the used alloys left at the end of the furnace or unused space in the furnace. The percentage of the waste is defined by the value of the sum of waste of each furnace, in kilograms, divided by the sum of the volume of all the scheduled furnaces. It should be noted that the furnace waste is not part of the objective function due to the company not knowing the costs inherent to this factor.

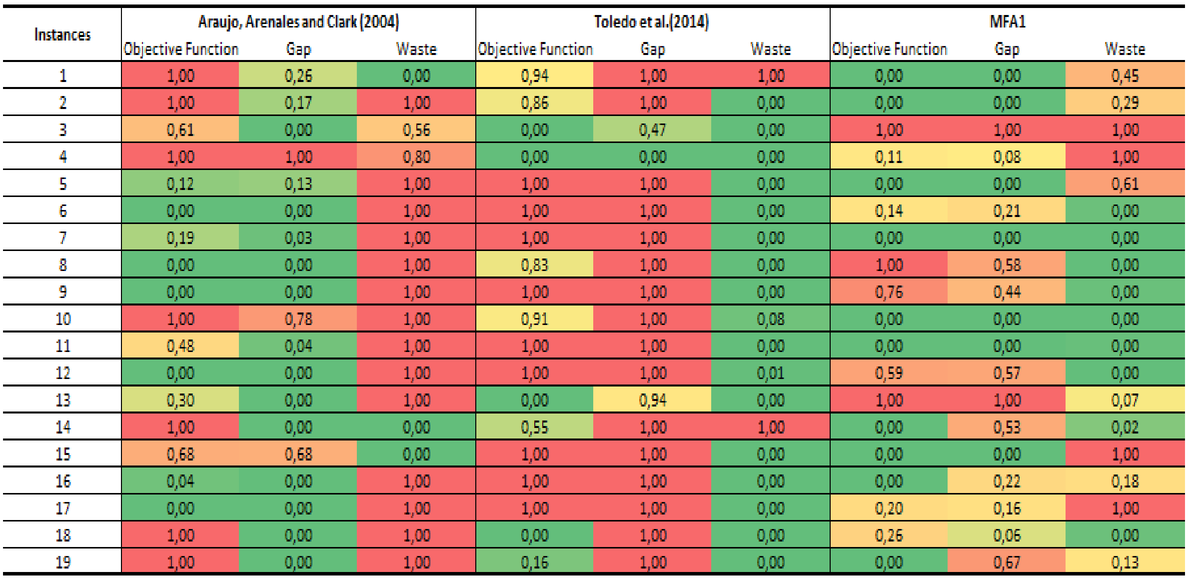
Table 6 - Waste for instances 1 to 19.

Instance	**Araújo, Arenales and Clark**		**Toledo et al. (2014)**		**MFA1**	
	Waste (kg / %)		**Waste (kg / %)**		**Waste (kg / %)**	
1	**191.05**	**1.68%**	191.60	1.68%	191.30	1.68%
2	956.10	6.45%	**616.10**	**4.25%**	716.10	4.91%
3	164.30	1.44%	**163.60**	**1.44%**	164.85	1.45%
4	452.85	2.38%	**452.25**	**2.38%**	453.00	2.38%
5	1019.90	8.94%	**400.50**	**3.72%**	779.60	6.99%
6	2039.45	10.73%	**1798.35**	**9.59%**	1799.45	9.59%
7	1018.90	8.94%	780.70	7.00%	**780.40**	**6.99%**
8	3558.50	18.73%	**1978.50**	**11.36%**	**1978.50**	**11.36%**
9	1019.20	8.94%	**778.25**	**6.97%**	779.20	6.98%
10	5268.70	27.73%	2648.70	16.17%	**2408.70**	**14.92%**
11	1627.80	8.57%	**1163.20**	**6.28%**	1164.40	6.29%
12	6973.95	36.70%	2067.35	14.62%	**2005.35**	**14.26%**
13	2626.35	13.82%	**1086.35**	**6.22%**	1201.65	6.84%
14	**42.80**	**0.23%**	167.30	0.88%	44.90	0.24%
15	8.95	0.05%	**4.50**	**0.02%**	2786.85	15.05%
16	2826.65	14.88%	**1946.00**	**10.79%**	2106.65	11.52%
17	1343.75	7.07%	**1030.65**	**5.57%**	1343.15	7.07%
18	1341.60	7.06%	**877.00**	**4.74%**	879.20	4.75%
19	6906.95	40.39%	**2366.95**	**18.85%**	2966.95	22.55%

Source: Authors (2021).

Considering the results presented in Tables 3 and 4, a simple scoring criterion was created, comparing all instances of each model, with values at "0" for the best result and "1", for the worst result. Figure 3 clearly shows which the best performance is for each instance. A color scale was defined to make it easier to interpret the results where red represents the worst-case scenario (closest to the value 1) and green, the best comparison scenario (0). It is worth noting that the model proposed by Araujo, Arenales and Clark (2004) works with a single furnace.

Figure 2 - Visual comparison of the results obtained for the test instances.



Instances	Araujo, Arenales and Clark (2004)			Toledo et al.(2014)			MFA1		
	Objective Function	Gap	Waste	Objective Function	Gap	Waste	Objective Function	Gap	Waste
1	1,00	0,26	0,00	0,94	1,00	1,00	0,00	0,00	0,45
2	1,00	0,17	1,00	0,86	1,00	0,00	0,00	0,00	0,29
3	0,61	0,00	0,56	0,00	0,47	0,00	1,00	1,00	1,00
4	1,00	1,00	0,80	0,00	0,00	0,00	0,11	0,08	1,00
5	0,12	0,13	1,00	1,00	1,00	0,00	0,00	0,00	0,61
6	0,00	0,00	1,00	1,00	1,00	0,00	0,14	0,21	0,00
7	0,19	0,03	1,00	1,00	1,00	0,00	0,00	0,00	0,00
8	0,00	0,00	1,00	0,83	1,00	0,00	1,00	0,58	0,00
9	0,00	0,00	1,00	1,00	1,00	0,00	0,76	0,44	0,00
10	1,00	0,78	1,00	0,91	1,00	0,08	0,00	0,00	0,00
11	0,48	0,04	1,00	1,00	1,00	0,00	0,00	0,00	0,00
12	0,00	0,00	1,00	1,00	1,00	0,01	0,59	0,57	0,00
13	0,30	0,00	1,00	0,00	0,94	0,00	1,00	1,00	0,07
14	1,00	0,00	0,00	0,55	1,00	1,00	0,00	0,53	0,02
15	0,68	0,68	0,00	1,00	1,00	0,00	0,00	0,00	1,00
16	0,04	0,00	1,00	1,00	1,00	0,00	0,00	0,22	0,18
17	0,00	0,00	1,00	1,00	1,00	0,00	0,20	0,16	1,00
18	1,00	0,00	1,00	0,00	1,00	0,00	0,26	0,06	0,00
19	1,00	0,00	1,00	0,16	1,00	0,00	0,00	0,67	0,13

Source: Authors (2021).

Analyzing the results presented in Figure 3 concerning the objective function value obtained, the MFA1 model achieved better results when compared to the other models in question. The smallest optimality gaps are observed in the model proposed by Araujo, Arenales and Clark (2004) for a single furnace. Considering the models with multiple furnaces, MFA1 showed a better objective function and optimality gap results than the model put forward by Toledo et al. (2014). Regarding furnace waste, the Toledo et al. (2014) model returned the best results. Thus, considering the company's need to use multiple furnaces, the good results obtained for the objective function, and the reasonable results obtained for the furnace waste, the MFA1 model was chosen as an application for the studied foundry.

4.2. Application of the MFA1 model in the foundry

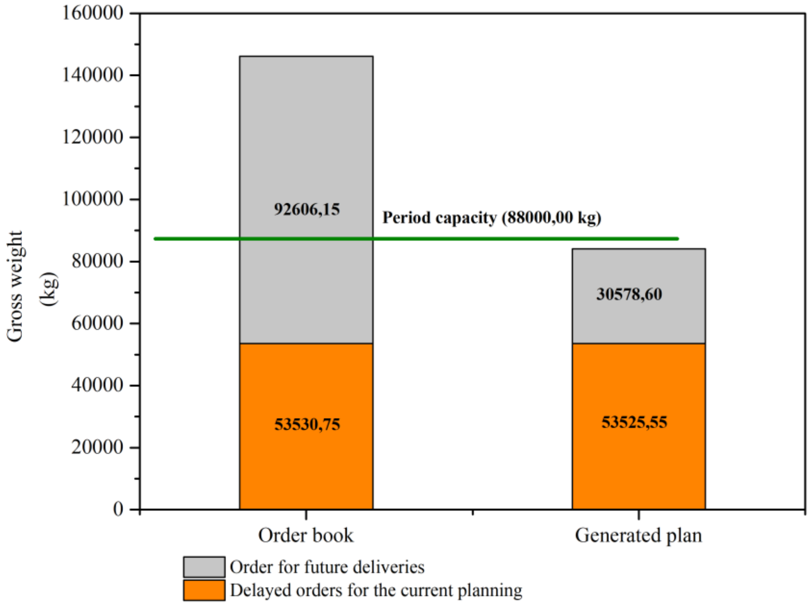
After defining the model to be adopted, the MFA1 model was solved using real data from the foundry, and the results obtained were compared with the current production plans practiced by the company. This section presents the results of the tests for the MFA1 model with the actual data collected at the company. For a broad comparison of the proposal, two scenarios were created:

1) A tactical production plan with twenty-two periods and five micro-periods (furnace loads) each; to verify the fulfillment of the order book and the possibility of generating a raw material requisition plan.
2) An operational production plan with five periods and five micro-periods (furnace loads); to verify the feasibility of the tactical planning.

It can be emphasized that the foundry does not visualize the first case in advance and the entire material request plan is made based on a less precise horizon. For comparison purposes, the foundry's operational plans for one month were surveyed.

The order book used to run the plan was collected from the company's PPC software. All orders available in the software up to the interview date with the company's employees were considered. There are 214 different items, totaling 146.1 tons of material to be melted. From this volume, approximately 19% were delayed at the start of planning (delivery planned for periods before the planning date) and approximately 18% had a delivery date for the planning month. Therefore about 53.53 tons of alloy were due for delivery in the current planning horizon. The remaining 92.60 tons of alloy are future orders, that is, they should be delivered in months after the current planning horizon. Figure 4 shows the quantity, in kilograms, of products demanded for the order book, considering the tactical planning with $T = 22$ periods, the available production capacity of $88000\ kg$ and the production plan generated by the MFA1 model.

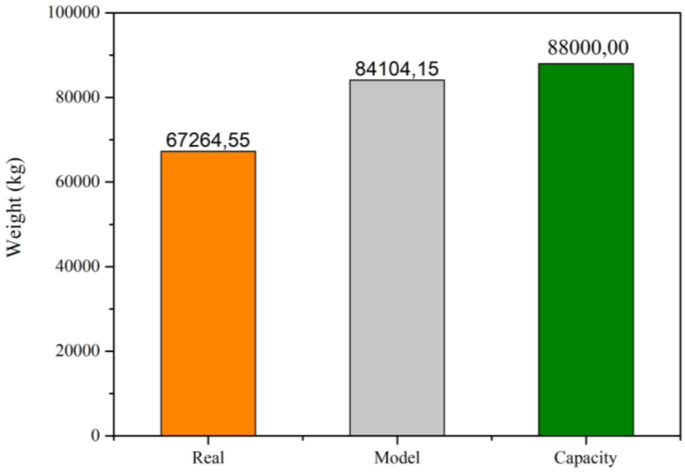
Figure 3 - Comparison of available order book and plan generated by the model in 22 periods.



Source: Authors (2021).

Considering a planning horizon with $T = 22$ periods, the MFA1 model practically eliminated delays and anticipated 33.02% of the future order book. The anticipation aims to reduce the furnace waste (occupation of 95.57% for the planned period) and is viable when holding costs are low. Figure 4 shows the scenario outlined in the plan compared to the available order book for the planning month. 99.99% of the backordered items were planned by weight. The unplanned items are four 1.3 kg parts that were left out of the tactical plan due to their low weight. Figure 5 illustrates the tactical plan generated compared to the sum of all operational plans made by the foundry during data collection. The same twenty-two periods were considered. A schedule that is 25.03% larger in casting volume than that performed by the foundry can be observed. This volume represents delayed items that were better allocated to the furnaces by the model, as well as anticipation of items, as illustrated in Figure 4. This scenario shows the unfeasibility of using the smaller capacity furnace. In some cases, holding costs may be lower than the setup cost for smaller furnaces.

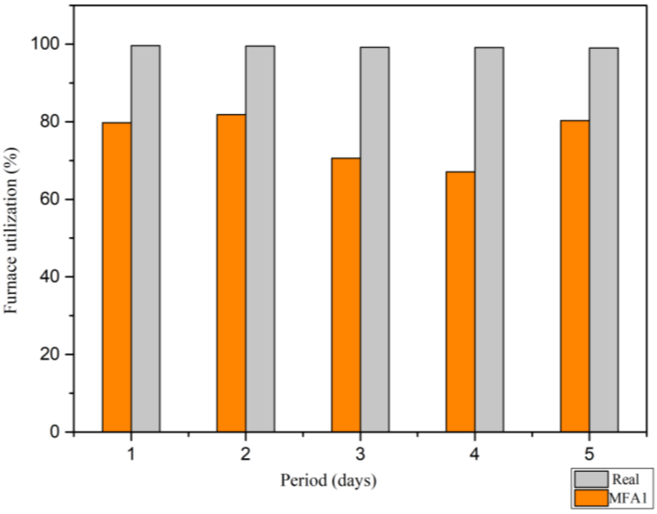
Figure 4 - Monthly furnace utilization comparison.



Source: Authors (2021).

Figure 6 compares furnace utilization between the production plan executed by the company and the one proposed by the MFA1 model. The regular utilization of the furnace (realized) can be observed at around 75.9% for the analyzed week. In comparison, the plan generated by the MFA1 model prioritized the anticipation of items and reached an average of 99.3% in the plans generated as a test. The difference in utilization found between the plan with twenty-two periods shown in Figure 5 (4.43%) and the operational plan with five periods shown in Figure 6 (0.7%) was expected. The model tends to regroup a larger volume of items precisely in a smaller planning period because it reduces the days available for production. By grouping the items for production better, the furnace waste was reduced.

Figure 5 - Comparison of furnace utilization for the operational plan.



Source: Authors (2021).

Finally, the delivery attendance of priority items (longest delay) was checked. There were no significant divergences between the current plan and the plan generated by the model. Due to the penalization of overdue items by the MFA1 model and the fact that the company already practices prioritization of overdue items, similar results were expected.

Both operational and tactical plans deliver important results for the foundry. The operational plan ensures the best furnace utilization with weekly updating of the order book. The tactical plan allows decision-makers to visualize risk situations and create more precise strategies.

4.3. Model validation questionnaire results

This section presents the results obtained from applying the experimental and operational validation questionnaires for the MFA1 model. Table 7 presents the answers obtained for Questionnaires 1 and 2.

Table 7 - Consolidated score of Questionnaires I and II.

	Experimental validation					**Operational validation**				
Questions	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
Mechanical engineer (trainee)	3	3	3	3	3	2	3	3	2	1
Mechanical engineer	3	3	3	3	3	2	3	3	3	1
PPC Leader	3	3	3	3	3	2	3	3	2	1
Average	3	3	3	3	3	2	3	3	2	1

Source: Authors (2021).

It can be observed that regarding Experimental Validation, those responsible for PCP consider it appropriate to use the MFA1 model to obtain a production plan that represents the company's reality and positively impacts the best use of the furnaces. The questionnaires also point out that a better definition of parameters generates more accurate results.

Regarding Operational Validation, the most highlighted point in the questionnaire refers to the cost of implementing a commercial package (Question 5). The employees responsible for PPC judge that the costs of implementing the model to determine the planning and sequencing of items and alloys are not compatible with the budget available by the company. As an alternative, it is suggested to use heuristics already consolidated in the literature or less robust packages but financially feasible. Question 1 shows observations regarding the model operationalization. However, it is the consensus of the team members that by generating standard bases and training the team, the difficulty of developing the plan is overcome.

Finally, the barriers signaled in Question 4 refer to the possibility of a high quantity of planned items occurring in a single furnace load, as mentioned previously. However, this does not make using the model unfeasible in practice because they are isolated events and items with many parts can be programmed manually.

5. DISCUSSIONS

The present work corroborates papers found in the literature regarding the mathematical model for lot-sizing and scheduling items in small foundries. Thus, as Araujo *et al.* (2008)

pointed out, the mathematical model can build good production plans in a computationally shorter time and with better use of the furnaces than the current methods practiced by the company.

As the MFA1 model aims at minimizing holding and delayed costs of items, an advantage of its application for the foundry is to generate a production plan for $T = 22$ periods. This plan can meet about 99% of the delayed items at the beginning of the production planning. This result is relevant since the company itself seeks to prioritize the production of delayed items due to costs incurred in the delay of orders and better organization of production planning. Additionally, it improves the company's relationship with customers.

Regarding optimality gaps, analyzing Table 5, it is noted that CPLEX achieved good solutions in solving the MFA1 model for instances 1 to 11. For these instances, we have an average optimality gap of 1.61%, indicating these solutions are close to optimal. For the larger data sets, represented by instances 12 to 19, CPLEX found it difficult to solve all three models tested. This result was already expected, as Duda and Stawowy (2018) pointed out when they noted that the use of heuristic or meta-heuristic algorithms is more appropriate for solving large instances.

Tonaki and Toledo (2010), Camargo *et al.* (2012), and Furtado *et al*. (2019) suggest the importance of reducing furnace waste for market foundries. The waste is caused by allocating alloys to be melted in smaller quantities than the furnace capacity. The MFA1 model showed an average waste of 8.20% of furnace capacity across all instances, which is slightly worse than the average 6.98% waste obtained by the model of Toledo et al. (2014). However, the MFA1 model obtained lower total costs in the objective function, which is why it was chosen and applied to the foundry under study.

However, the difficulty presented by the company when determining delay and inventory costs should be mentioned. These costs were determined subjectively for this work. The cost parameters do not invalidate the model. The people involved in the validation process stated that the parameters proposed by the literature (inventory and delay costs) assist in decision making and can be used until the actual foundry cost survey.

Limiting production planning to two days in advance is the minimum acceptable by the company for the schedule to proceed without major interferences. Increasing this planning capacity can reduce the risks of mold shortages, delays, and waste furnaces. Moreover, the plan will provide important information to those responsible for procurement and communication with customers, as well as predict the billing for the period. Another advantage is the elimination of idleness in the core shop and mold-making sectors.

Due to the possibility of a sudden increase in demand, it is considered interesting to extend the model to plan the use of parallel furnaces. This situation requires dedicated analysis, due to the increase in demand for contracted energy and, consequently, the increase in setup costs. This alternative is valid to reduce delays or increase production capacity without large investments in capital goods.

6. CONCLUSIONS

This paper presents an extension to a literature model for the lot-sizing and scheduling problem with multiple alternate furnaces in a small foundry. The contribution to the literature is in the mathematical modeling of a real problem little addressed in previous works. In addition, comparative tests between the proposed model and two others existing in the literature were performed to define if the model could represent the company's production planning. The comparison between models that consider multiple non-simultaneous furnaces brings new contributions to select a model that best fits the characteristics presented in the literature for the problem.

Operational plans were generated with a five-day planning horizon, using actual data provided by the company's Production sector. The results obtained using the model determined a production plan with lower costs than those currently practiced by the company. Moreover, better utilization of the furnaces' capacity was provided. In order to assist the decision-making process at the managerial level, tactical plans were generated, with a planning horizon of twenty-two days. The tactical plan anticipates the production of demanded items leading to better utilization of the foundry's installed capacity.

To validate the use of the proposed model, two questionnaires were conducted with the people responsible for the company's Production sector. The respondents had to rate their perceptions regarding the results obtained by the model. It was concluded that the model represents the lot-sizing and scheduling problem for the studied foundry. However, implementing optimization software is not economically feasible for the company.

In the future, we suggest carrying out a precise survey of costs contained in the objective function and other operating costs. A lack of knowledge of these costs is common in the process industry. Thus, the reliability of the production plans generated by the model will be increased. Another suggestion for the model application is the inclusion of constraints to regulate the number of items to be produced per furnace load. This point was raised during the questionnaire and may be a limiting factor due to the production capacity of boxes. This suggests a model extension to multiple levels, considering the foundry planning integrated with the other productive sectors such as core shop, mold making and finishing.

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Appendixes

Appendix I - Questionnaire 1: Experimental validation

Questionnaire 1 - Experimental validation		
Position		
Main functions		
Time on the job		
Schooling		
Score: 1 – I do not agree 2 – I agree with restrictions 3 – I completely agree		
1 - Do you agree that using the proposed model leads to a better perception of the impact of production planning?		
Score: 1,2 or 3?		Please, explain why.
2 - Do you think that changing the parameters in the model can represent operational reality?		
Score: 1,2 or 3?		Please, explain why.
3 - Can the presented model be used to size and schedule the items and formulate production plans?		
Score: 1,2 or 3?		Please, explain why.
4 - Does the model faithfully present the lot sizing and production planning problem?		

Score: 1,2 or 3?		Please, explain why.
5 - Is the result useful and can it contribute, in any way, to developing production plans?		
Score: 1,2 or 3?		Please, explain why.

Source: Authors (2021).

Appendix 2: Questionnaire 2 - operational validation

Questionnaire 2 - Operational validation		
Position		
Main functions		
Time on the job		
Schooling		
Score: 1 – I do not agree 2 – I agree with restrictions 3 – I completely agree		
1 - Do you consider the model's operationalization simple?		
Score: 1,2 or 3?		Please, explain why.
2 - Is the model useful to formulate production plans in synergy with the company's strategies?		
Score:		Please, explain why.

1,2 or 3?		
3 - Does data input and model execution provide a timely solution for utilizing the production plan?		
Score: 1,2 or 3?		Please, explain why.
4 - Are the results obtained in line with the premises and observations prior to production planning?		
Score: 1,2 or 3?		Please, explain why.
5 - Is the cost to implement an optimization system coherent with the obtained result and is it tangible for the company?		
Score: 1,2 or 3?		Please, explain why.

Source: Authors (2021).